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COMMENT

Extrapolation of transfer matrix data for percolation and lattice animals by the Romberg–Beleznay algorithm

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Abstract. Using Beleznay's modified Romberg algorithm we re-analyse the transfer matrix data for the two-dimensional problems of Derrida and Stauffer. We get for percolation: $p_c = 0.592\,73(6)$ for the threshold of the square site problem, $\nu = 1.3330(7)$ for the correlation length exponent and $\omega = 1.9(3)$ for the correction exponent; in the case of lattice animals $x_c = 0.246\,152(4)$ (square lattice), $x_c = 0.192\,94(1)$ (triangular lattice critical fugacity) and $\nu = 0.640\,62(10)$ (radius exponent).

The transfer matrix method leads to very accurate results on finite two-dimensional strips, which can be used to obtain approximate values of critical parameters by extrapolation (Nightingale 1982). The accuracy of the estimates depends, besides the number of data used and their relevant number of digits, on the speed of the convergence, and therefore on the method of extrapolation too. Recently Beleznay (1985) proposed a so-called modified Romberg algorithm to extrapolate finite size scaling data efficiently. The aim of the present comment is to use Beleznay's method to evaluate the new transfer matrix data for two-dimensional percolation and lattice animal problems calculated by Derrida and Stauffer (1985, hereafter referred to as DS).

In Beleznay (1986) it is shown how to order the input data and form a Romberg-type triangular array. In the case of critical finite size behaviour the critical exponent is a parameter of this array. The convergence along the diagonals is much faster than in the original data and therefore the last value in the principal diagonal can be regarded as the best estimate. Moreover, if the exponent is known, an error of this estimate can also be given. Beleznay (1986) distinguished three different cases.

(i) Method A. If neither the limiting value nor the exponent is known, then a minimisation of the expression of the error gives the approximate values.

(ii) Method B. If the limiting value is known the exponent can be determined by forcing the 'estimate' to give the correct value.

(iii) Method C. If the exponent is known the estimated limiting value can be calculated and the accuracy determined.

DS used the transfer matrix technique for three different geometries: square lattice with transfer direction along a lattice axis (S1); square lattice with transfer direction along the diagonal (S2) and triangular lattice (T).

For percolation we first applied method A, forgetting for a while that p_c is known for geometry T ($=\frac{1}{2}$). Using all data and even-odd analysis, omitting the first 1, 2 and 3 terms, we got the impression that, fortunately, the data for T are the most reliable,

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i.e. in this case the result was not very much changed. By simple minimisation of the error and using all data we obtain $p_c = 0.500\,004$ and for the critical exponent ρ, which describes the finite size scaling of the p_c data, the value $\rho = 2.6$ was found. Moreover the data for S1 were found to be the worst, in accordance with DS (D Stauffer, private communication).

Then we applied method B in order to get an estimate for ρ and this led to $\rho = 2.649$. Now method C could be used for the geometries S1 and S2: $p_c = 0.592\,72(25)$ (S1) and $0.592\,728(55)$ (S2). Again, the data for S1 turn out to be quite poor. The errors are in the last digits and calculated from the Romberg error formula (Beleznay 1986), i.e. the error for S2 is about five times smaller than for S1. If one could believe that the nice coalescence of the results for S1 and S2 is not accidental, the estimated value for p_c for the square site percolation could be given as $p_c = 0.592\,73(1)$.

In order to extrapolate the data for the critical exponent ν, we have to use the relation $\rho = \omega + 1/\nu$ (Derrida and de Seze 1982), ω describing the finite size scaling of the ν data. Since we know already the estimate for ρ, the limiting value of ν can be calculated self-consistently. We get $\nu = 1.3330(7)$ which fits well with the 'exact' value of $\frac{4}{3}$. Repeating the same calculation for the geometries S1 and S2, one always gets results which are compatible with $\frac{4}{3}$, but with much larger errors. On the other hand, method A can again be used: then the exponent ω is also determined. From these calculations we have the estimate $\omega = 1.9(3)$.

We also evaluated the data for the exponent η. Since there is no *a priori* known exponent which describes the finite size scaling of this series, we applied method A. From geometry T $\eta = 0.208\,37$ (which should be compared to the 'exact' 0.208 333) and the correction exponent = 1.95 which suggests the possibility that the finite size behaviour of the series for ν and η are governed by the same correction exponent ω. Repeating this calculation for the geometries S1 and S2 we get similar, but again worse, results.

For the lattice animals, no solution is known; therefore method A is to be used. Again in accordance with DS we have found that one of the geometries, here S1, is much superior to the rest. We get for the critical fugacity $x_c = 0.246\,152(2)$, where the error is now estimated by using different samplings from the original set of data. The exponent ρ, describing finite size scaling, is about 2.8. The similar analysis of the radius exponent leads to $\nu = 0.6406(1)$ with the correction exponent $\omega = 2.1(3)$; the relation $\rho = \omega + 1/\nu$ seems to be violated. For geometry S2 two minima were first found by method A. However, one of them could be excluded by sampling. From these data $x_c = 0.246\,156(6)$, in good agreement with S1, but for the exponent $\nu = 0.6414(4)$, which is in conflict with S1. Similarly, from geometry T $\nu = 0.6414$. These discrepancies were already recognised by DS. For the critical fugacity on the triangular lattice we get $x_c = 0.192\,94(1)$, a definitely higher value than that obtained by DS.

In conclusion, we have analysed the DS data by the modified Romberg algorithm proposed by Beleznay (1986). Generally, the method leads to results of the same or sometimes higher accuracy as the analysis by DS. The method is easy to carry out, excludes much subjectivity and in some cases (method C) it gives a good estimate of the errors. Therefore it seems to be a method at least competitive with the other ones and possibly better.

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